CHAPTER FOUR

VARIATION

Introduction:

This shows the relationship which exists between two variables or quantities.
- For example we may choose to consider the relationship between the number of gallons of petrol used by a travelling car and the distance travelled.
- This relationship or variation will be that, the greater the distance travelled, the greater will be the number of gallons of petrol used.

- Variation truly speaking is the same as proportion.

Types:

- Basically there are three types and these are: :

1. Direct variation.

2. Inverse variation.

3. Partial variation.

NB: Associated with each of these is what certain individuals refer to as joint variation

DIRECT VARIATION :

-This may also be referred to as direct proportion:

-Two variables or quantities such as x and y are said to vary directly, or are directly proportional if

1. both x and y increases at the same time e.g

X	2	3	4	5	6	7
Y	4	8	12	16	20	24

2. both values of x and y decrease at the same time. i.e X decrease as y decreases. e.g.

X	100	80	60	40	20
Y	10	8	6	4	2

Now consider two variables x and y. If x varies directly as y, then we write
$x \propto y$, i.e x is directly proportional to y.

From $x \propto y$, in order to remove the proportional sign, we must introduce a constant,

i.e if $x \propto y$, $\Longrightarrow$ x = ky, where k = a constant, which may be referred to as the proportionality constant or the constant of proportionality.

Also if M varies directly as R, then $M \propto R \Longrightarrow M = K.R$, where K is a constant.

Q1. Two variables m and v are such that m varies directly as v. Given the constant as 10, calculate m when v = 20.

Solution

Since m varies directly as v

$\Longrightarrow m \propto v, \Longrightarrow m = kv$

Where k is a constant.

Since k = 10 $\Longrightarrow$ m = 10v.

When v = 20 $\Longrightarrow$ m = (10)(20) = 200.

Q2. The population, P of a nation is directly proportional to the birth rate, R. If the constant is 20, determine the population in million if R = 5.

Solution.

P varies directly as R

$\Longrightarrow p \propto R \Longrightarrow p = KR$,

Where k is a constant. Since k = 20 $\Longrightarrow$ p = 20 R.

When R = 5 $\Longrightarrow$ P = (20) (5) = 100

The population, P = 100 million.

Q3. The velocity v of a car is directly proportional to its mass m squared. Given the constant as 5 and the mass of the car as 60kg, calculate the velocity in m/s.

Solution.

V is directly proportional to m squared $\Longrightarrow v \propto m^2$,

$\Longrightarrow v = km^2$, where k is a constant.

Since k = 5 => v = $5m.^2$

When m = 60 ⟹ v = $5(60)^2$,

⟹ v = 5 (3600) = 1800,

=> V = 18000 m\s.

Q4. The circumference of a circle varies directly as it radius cubed. Determine the circumference in cm, if the radius is 3cm. Take the proportionality constant to be 4.

Solution

Let c = the circumference and r = the radius.

Since the circumference varies directly as the radius cubed

⟹ $C \propto r^3$ ⟹ $c = kr^3$,

where k = the constant. Since r = 3cm and k = 4

⟹ c = $(4)(3)^3 = (4)(27) = 108.$

The circumference = 108cm.

Q5. The density of a material is directly proportional to the square root of its mass. Given the variation constant as 5 and the mass as 25g, determine the density in g cm^{-3}.

Let d = density and m = mass.

Since the density is directly proportional to the square root of the mass

⟹$d \propto \sqrt{m}$,

⟹ $d = k.\sqrt{m}$, where k = the constant. Since k = 5 and m =25 ⟹$d = 5.\sqrt{25}$, ⟹ d = (5) (5) = 25,

⟹ d = 25 cm^{-3}

Q6. The speed of a vehicle varies directly as it mass. The speed also varies directly

as its length. Find the speed in km/h, if the car is of mass 120kg and it is 50 m long. Take the variation constant to be 10.

Solution.

Let s = speed, m = mass and *l* = length.

1. The speed varies directly as the mass ⟹ $s \propto m$………….(1)

11. Also the speed varies directly as the length $\Rightarrow s \propto l$.........*(2)*

From 1 and 2 i.e $s \propto m$ and $s \propto l$

$\Rightarrow s \propto m.l.$ Removing the proportion sign

$\Rightarrow s = k.m.l.$

Bu since k = 10, m = 120 and l = 50

$\Rightarrow s = (10)(120)(50) = 60{,}000$

$\Rightarrow s = 60{,}000$ km/h.

Q7. Three quantities, R, Q and M are such that R varies directly as Q. R also varies directly as M. If the proportionality constant is 20, find R when Q = 1 and m = 7.

Solution

1. R varies directly as Q

$\Rightarrow R \propto Q$.

11. Also R varies directly as M.

$\Rightarrow R \propto M$.

From $R \propto Q$ and $R \propto M$

$\Rightarrow R \propto Q.\,M \Rightarrow R = KQ.M.$

But since Q = 1, M = 7 and k = 20

$\Rightarrow R = (20)(1)(7) = 140$,

$\Rightarrow R = 140$.

Q8. The velocity of a car varies directly as its mass squared. It also varies directly as the length of the car. Given that the car has a length of 10m and a mass of 30kg, calculate its velocity in km/h. Take the proportionality constant as 200.

Solution

Let v = velocity, m = mass and l = length.

1. Since the velocity varies directly as the mass squared

$\Rightarrow$ v ∝ m^2.

2. Also since the velocity varies directly as the length

$\Rightarrow$ V ∝ *l*.

From 1. V ∝ m^2 and from 2. V ∝ *l*,=> $v \propto m^2 l \Rightarrow v = km^2 l$.

But m = 30, k = 200 and l = 10 .

From v = $km^2l \Rightarrow v = (200)(30)^2(10)$,

$\Rightarrow$ v = (200) (900) (10),

$\Rightarrow$ v = 18 000 00 km/h.

Q9. The quantities Q, L and Z are such that L varies directly as Q cubed as well as Z. Given that L = 160 when Q = 2 and Z = 5, calculate the variation constant.

Solution

1. L varies directly as Q cubed $\Rightarrow L \propto Q^3$.

11. L varies directly as Z $\Rightarrow$ L ∝ Z .

Now from $L \propto Q^3$ and L ∝ Z

$\Rightarrow L \propto Q^3. Z, \Rightarrow L = K.Q^3. Z$.

But since L= 160, when Q = 2 and Z = 5,$\Rightarrow$ L= k $(2)^3(5)$,

=> 160 = K(8)(5) $\Rightarrow$160 = 40K,

$\Rightarrow$ K = 160/40 = 4.

The constant = 4.

10. The distance d covered by a vehicle varies directly as its length, *l* cubed as well as the square root of its mass m. Given that when m = 9kg,*l* = 2m and d = 120km,

1. determine the value of the constant.

11. deduce an expression for d in terms of m, l and k, where k is the constant

111. determine the distance travelled by the car in kilometers, when it is 40m long and has a mass of 60kg.

Solution

1.Since d varies directly as l cubed $\Longrightarrow$ d $\propto l^3$

11. Since d also varies directly as the square root of m

$\Longrightarrow$ d $\propto \sqrt{m}$.

Now since d $\propto l^3$ and d $\propto \sqrt{m}$,

$\Longrightarrow$ d $\propto l^3.\sqrt{m}$ => d = k.l^3. $\sqrt{m}$..

But d = 120 when l = 2 and m = 9, and since d = k $l^3\sqrt{m}$,

$\Longrightarrow$120 = k$(2)^3(\sqrt{9})$ = k(8)(3) = 24k,

$\Longrightarrow$120 = 24k $\Longrightarrow$ k = $^{120}/_{24}$ = 5.

$\therefore$ K = 5.

11. We have already noted that d = $kl^3\sqrt{m}$.

To deduce an expression for d in terms of k, l and , m, we only substitute the value of the constant (i.e. 5) into the just written expression.

From d = $kl^3 \sqrt{m}$ $\Longrightarrow$d = $5l^3\sqrt{m}$.

111. When l = 40m and m = 60kg,

$\Longrightarrow$ d = 5 $l^3 \sqrt{m}$ $\Longrightarrow$d = $5(40)^3 \sqrt{60}$,

$\Longrightarrow$ d = 5 (64000) (7.7),

$\Longrightarrow$ d = 2,464 000.